1) Normal Distribution

- In a Normal distribution, 7% of the items are under 35 and 89% are under 63. Find the mean and standard deviation of the distribution.

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In a normal distribution 7% of the items are under 35 and 89% under 63. Determine the mean of variance of the destribution.

Let μ = mean

σ = Standard Deviation.

Given: $P(x < 35) = 7\%$

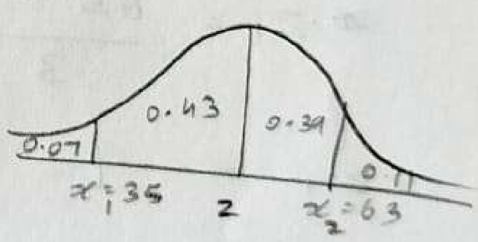
$P(x < 35) = 0.07$

$P(x < 63) = 89\%$

$= 0.89$

$P(x > 63) = 1 - 89\%$

$= 0.11$



$P(x < 35)$

$x_1 = 35$

$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{35 - \mu}{\sigma}$

$A(0.43) = \frac{35 - \mu}{\sigma}$

$1.48 = \frac{35 - \mu}{\sigma}$

$35 = 1.48(\sigma) + \mu$

$P(x > 63)$

$x_2 = 63$

$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{63 - \mu}{\sigma}$

$A(0.39) = \frac{63 - \mu}{\sigma}$

$1.23 = \frac{63 - \mu}{\sigma}$

$63 = 1.23(\sigma) + \mu$

Solving, $\mu = 50$, $\sigma = 10.332$.

2) Population Statistics

- A population consists of five numbers 2, 3, 6, 8, and 11. Consider all possible samples of size two which can be drawn with replacement from this population.
  a) The mean of the population.
  b) The standard deviation of the population.
  c) The mean of the sampling distribution of means.
  d) The standard deviation of the sampling distribution of means.

The total number of sampling with replacement : N^n

$$S = N^n$$

Given : $N = 5$, $n = 2$

$$S = 5^2 = 25$$

i) Mean of the population : $\mu = \frac{\Sigma x_i}{N}$

Given : $x_i = 2, 3, 6, 8, 11$.

$$= \frac{2+3+6+8+11}{5} = \frac{30}{5}$$

$$= 6.$$

ii) Standard deviation of the population (N)

$$\sigma = \sqrt{\frac{(x_i - \mu)^2}{N}}$$

$$= \sqrt{\frac{(2-6)^2 + (3-6)^2 + (6-6)^2 + (8-6)^2 + (11-6)^2}{5}}$$

$$= \sqrt{\frac{54}{5}}$$

$$= 3.2863$$

iii) The mean of sampling distribution of Means.

$$\left\{\begin{matrix} (2,2),(2,3),(2,6),(2,8),(2,11) \\ (3,2),(3,3),(3,6),(3,8),(3,11) \\ (6,2),(6,3),(6,6),(6,8),(6,11) \\ (8,2),(8,3),(8,6),(8,8),(8,11) \\ (11,2)(11,3)(11,6),(11,8),(11,11) \end{matrix}\right\} \text{Sampling distribution}$$

Mean of Sampling distribution of Means :

$$= \frac{150}{25} = \frac{\frac{2+2+2+3+2+6+2+8+2+11+3+2+3+3+\dots}{2}}{25}$$

$$= 6$$

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iv) SD of sampling Destribution of Means

$$\sigma^2 = \frac{(x_i - \mu)^2}{N^n}$$

$$= \frac{\left(\frac{2+2}{2} - 6\right)^2 + \left(\frac{2+3}{2} - 6\right)^2 + \left(\frac{2+6}{2} - 6\right)^2 + \cdots}{25}$$

$$\sigma^2 = 6$$

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$$\sigma = 2.32.$$

3) Marks in Mathematics

- Marks obtained in Mathematics by 1000 students are normally distributed with a mean of 78% and a standard deviation of 11%. Determine:
  a) How many students got marks above 90%.
  b) What was the highest marks obtained by the lowest 10% of the students.
  c) Within what limits did the middle 90% of the students lie.

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The marks obtained in Math By 1000 students is normally distributed with mean 78% and ~~x~~ Standard deviation 11%. determine i) How many students got ~~abo~~ above 90%.

ii) What was the heighest marks obtained by the lowest 10% of the student.

iii) with in what limits did the middle of 90% of the students lie.

Solution: Mean = 78% = 0.78

S.D = 11% = 0.11

i) $P(X > 90\%)$

Let $x_1 = 0.90$

$z_1 = 0.9$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{0.9 - 0.78}{0.11}$$

$$z_1 = 1.09$$

Hence the no. of students with marks more than 90%

$\Rightarrow P(X > 90\%) =$ ~~0.5 P(Z1~~

$0.5 - A(z_1)$

$= 0.5 - 0.3623$

$=$ ~~0.113~~ 0.1377

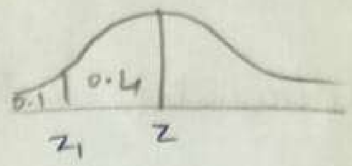
No. of Students $= P(X > 90\%) \times 1000$

$= 0.1377 \times 1000$

$= 138.$

ii)

$P(0.1)\ 0.5 - A(0.4)$



$$P(0.1) = 0.5 - A(-1.28)$$

$$z_1 = -1.281$$

$$z_1 = \frac{x_1 - \mu}{\sigma}$$

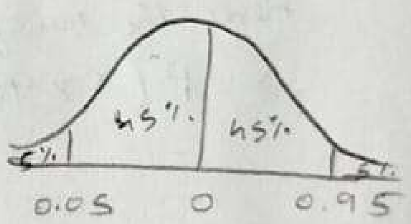
$$-1.281 = \frac{x_1 - 0.78}{0.11}$$

$$x_1 = 0.6392$$

Hence, Height mark obtained at bottom 10% of the students is 63.92%.

iii) Middle 90% corresponding to 0.9 area leaving 0.05 on both sides. Then the correspong z's are

$$P(5\% \le x \le 95\%) = P(0.05 \le z_1 \le 0.95)$$



$$\therefore z_1 = -1.645$$

$$z_2 = +1.645$$

$$z_1 = -1.645 \qquad z_2 = 1.645$$

$$z_1 = \frac{x_1 - \mu}{\sigma} \qquad z_2 = \frac{x_2 - \mu}{\sigma}$$

$$x_1 = 0.5985 \qquad x_2 = 0.9615$$

$\therefore$ Limits are are 59% to 96%.

HW In a sample of 1000 cases the mean of a certain test is 14 and Standard deviation 2.5. Assuming the distribution to be normal. Find i) How many students between 12 and 15.

4) Probability Calculation

- A random sample of size 64 is taken from an infinite population having a mean of 45 and a standard deviation of 8. What is the probability that x will be between 46 and 47.5.

4) Given:

$n = 64$ (large Sample)

$\mu = 45$ (Population)

$\sigma = 8$ "

To Find:

$P(46 < x < 47.5)$

Let $x_1 = 46$

$x_2 = 47.5$

For Test Hypothesis of a

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Single Mean - Large Sample:

$(\because \sigma$ is given)

Test Statistic: $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

Let $P(46 < x < 47.5) = P(z_1 < Z < z_2)$

$$z_1 = \frac{x_1 - \mu}{\sigma/\sqrt{n}} \qquad z_2 = \frac{x_2 - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{46-45}{8/\sqrt{64}} \qquad = \frac{47.5 - 45}{8/\sqrt{64}}$$

$$= \frac{1}{\cancel{8}/\cancel{8}} \qquad = \frac{2.5}{\cancel{8}/\cancel{8}}$$

$$z_1 = 1 \qquad z_2 = 2.5$$

$$P(46 < x < 47.5) = P(z_1 < z < z_2)$$

$$= P(1 < z < 2.5) \qquad = P(z_1 < z < z_2)$$

$$= |A(2.5) - A(1)| \qquad = |A(z_2) - A(z_1)|$$

$$= |0.49379 - 0.34135|$$

$$= 0.15244$$

$$\therefore P(46 < x < 47.5) = 0.15244$$

5) Sample Analysis

- A sample of 64 students has a mean weight of 70 kgs. Can this be regarded as a sample from a population with a mean weight of 56 kgs and a standard deviation of 25 kgs?

5) Given:

$n = 64$

Sample $\bar{x} = 70$ kg

Population $\mu = 56$ kg

" $\sigma = 25$ kg

To find:

H_0 : Null Hypothesis

$\mu = 56$ kg

H_1 : Alternative

$\mu \neq 56$ kg

Let level of significance = 5% (Two Tails)

Test Statistic: $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$$z = \frac{70 - 56}{25/\sqrt{64}} = \frac{14}{25/8} = 4.48$$

$Z_\alpha = 1.96$ (5% (two Tails) Level of Significance)

$|z| > Z_\alpha$

H_0 is Rejected.

6) Probability Calculation

- A normal population has a mean of 0.1 and a standard deviation of 2.1. Find the probability that the mean of a sample of size 900 will be negative.

6.) Given:

Sample $n = 900$

Population $\mu = 0.1$

$\sigma = 2.1$

To Find:

$\cancel{P(\mu < 0)}$

$P(\bar{x} < 0)$ Sample mean $\bar{x}$

Let $x_1 = 0$

$$\because Z = \frac{x_1 - \mu}{\sigma / \sqrt{n}}$$

$$Z_1 = \frac{0 - 0.1}{2.1/\sqrt{900}} = \frac{-0.1}{2.1/30}$$

$$Z_1 = -1.42857$$

$$P(\bar{x} < 0) = P(Z_1 < 0)$$

$$0.5 - |A(Z_1)|$$

$$= 0.5 - |A(-1.4285)|$$

$$= 0.5 - 0.42344$$

$$= 0.07656$$

$$\therefore P(\bar{x} < 0) = 0.07656$$

7) Hypothesis Testing

- In a sample of 1000 people in Karnataka, 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this state at a 1% level of significance?

7) Given:

$n = 1000$

No. of ~~wheat~~ rice eaters = 540

No. of wheat eaters = 1000 − 540
= 460

To Find:

H_0: $P = 500/1000$
at 1% significance
H_1: $P \neq 500/1000$

$P = 0.5$

Let $p = \frac{540}{1000} = 0.54$

$Q = 1 - P$
$= 0.5$

$q = 1 - p = 0.46$

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Test of Significance for Single Proportion

Test Statistic: $Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$

$$= \frac{0.54 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{1000}}}$$

$Z = 2.5298$

$Z_\alpha = 2.58$ 1% level of significance

$|Z| < Z_\alpha$ H_0 Approved is it?

8) Confidence Interval

- A sample of size 10 was taken from a population with a standard deviation of 0.03. Find the maximum error with 99% confidence.

A sample of size 10 was taken from a population S.D of sample is 0.03. Find the maximum error with 99% confidence

Sol:- Given standard deviation $S = 0.03$

$$n = 10$$

maximum error $E = t_{\frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}}$

$t_{\frac{\alpha}{2}}$ for 99% of $\nu = 9$ d.f $= 3.25$

$$\therefore \ E = 3.25 \times \frac{0.03}{\sqrt{10}}$$

$$E = 0.0325.$$

9) Confidence Interval (Repeat of 8)

- A sample of size 10 was taken from a population with a standard deviation of 0.03. Find the maximum error with 99% confidence.

10) Population Analysis

- A sample of 900 members has a mean of 3.4 cms and a standard deviation of 2.61 cms. Is this sample taken from a large population of mean 3.25 cm and standard deviation 2.61 cms? If the population is normal and its mean is unknown, find the

95% fiducial limits of the true mean.

10) Given: $n = 900$

$\bar{x} = 3.4$ cm

$s = 2.61$ cm

$\mu = 3.25$ cm

$\sigma = 2.61$ cm

To find:

Confidance Interval

H_0: Confidance $= 95\% \Rightarrow Z_\alpha = 1.96$

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{3.4 - 3.25}{2.61 / \sqrt{900}}$$

$$= \frac{\cancel{0.2}\ 0.15}{2.61/30} = 1.72414$$

$$|Z| < Z_\alpha$$

H_0: rejected.

$$\text{Confidance Interval} = \bar{x} \pm Z \cdot \frac{\sigma}{\sqrt{n}}$$

$$= \left(3.4 - 1.724\left(\frac{2.61}{\sqrt{900}}\right),\ 3.4 + 1.724\left(\frac{2.61}{\sqrt{900}}\right)\right)$$

$$= (3.25,\ 3.55)$$

11) Markov Chain

- The transition probability matrix of a Markov chain {Xn} ; n = 1, 2, 3... having three states 1, 2, and 3 is P = (
  0.1 0.5 0.4
  0.6 0.2 0.2

0.3 0.4 0.3

) and the initial distribution is P(0) = (0.7, 0.2, 0.1). Find:

a) P{X2 = 3}.

b) P{X3 = 2, X2 = 3, X1 = 3, X0 = 2}.

distribution is $p^{(0)} = (0.7, 0.2, 0.1)$

find (i) $P\{X_2 = 3\}$ (ii) $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$.

sol :- G.T the TPM of Markov chain is

$$P = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix} \end{array}$$

we have $P(X_0 = 1) = 0.7$, $P(X_0 = 2) = 0.2$

$P(X_0 = 3) = 0.1$

$$P^2 = P * P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix} \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1.43 & 0.31 & 0.26 \\ 0.24 & 0.42 & 0.34 \\ 0.36 & 0.35 & 0.29 \end{bmatrix}$$

(i) $P(X_2 = 3) = \sum_{i=1}^{3} P\{X_2 = 3 / X_0 = i\} \; P(X_0 = i)$

$= \sum_{i=1}^{3} P_{i3}^{(2)} \; P(X_0 = i)$

$= P_{13}^{(2)} P(X_0 = 1) + P_{23}^{(2)} P(X_0 = 2) + P_{33}^{(2)} P(X_0 = 3)$

$= 0.26\,(0.7) + 0.34\,(0.2) + 0.29\,(0.1)$

$= 0.279$

(ii) $P\{X_3 = 2,\ X_2 = 3,\ X_1 = 3,\ X_0 = 2\}$

$P(X_3 = 2 / X_2 = 3) \; P(X_2 = 3 / X_1 = 3) \; P(X_1 = 3 / X_0 = 2) \; P(X_0 = 2)$

$P_{32}^{(1)} \cdot P_{33}^{(1)} \cdot P_{23}^{(1)} \; 0.2$

$= 0.4 \times 0.3 \times 0.2 \times 0.2$

$= 0.0048$

12) Hypothesis Testing

- Nicotine contents in milligrams in two samples of tobacco were found to be as follows:
  Sample A: 24 27 26 21 25 -
  Sample B: 27 30 28 31 22 36
  Can it be said that the two samples have come from the same normal population?

12) Given:

To Find:

A 24 27 26 21 25

H_0 $\mu_A = \mu_B$

B 27 30 28 31 22 36

Let $\alpha = 0.05$

$n_A = 5$ $n_B = 6$

$$\bar{x}_A = \frac{24+27+26+21+25}{5} \qquad \bar{x}_B = \frac{27+30+28+31+22+36}{6}$$

$$\frac{123}{5} = 24.6 \qquad = \frac{174}{6} = 29$$

$$\bar{S}_A = \sqrt{\frac{(24-24.6)^2+(27-24.6)^2+\dots}{5}} \qquad \bar{S}_B = \sqrt{\frac{(27-29)^2+(30-29)^2+\dots}{6}}$$

$$\bar{S}_A = 2.059 \qquad \bar{S}_B = 4.2426$$

T static: $$t = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{\frac{S_A^2}{n_A} + \frac{S_B^2}{n_B}}}$$

$$= \frac{24.6-29}{\sqrt{\frac{2.059^2}{5} + \frac{4.24^2}{6}}} = -2.243$$

Degree of freedom = $df = (nA - 1) + (nB - 1)$

$= 9$

at $\alpha = 0.05$ with 9 degree of freedom

$t_\alpha = 2.306$

$|t| > t_\alpha$

H_0. Reject is it ?

13) Markov Chain Gambling

- A gambler has Rs. 2. He bets Re. 1 at a time and wins Re. 1 with a probability of 1/2. He stops playing if he loses Rs. 2 or wins Rs. 4.
  a) What is the transition probability matrix of the related Markov Chain?
  b) What is the probability that he has lost his money at the end of 5 plays?
  c) What is the probability that the game lasts more than 7 plays?

(5) A gambler has a ₹2. He bets 1 at a time and wins rupee 1 with a prob 1/2. He stops playing if he loses 2 Rs or wins 4 Rs

(a) what is the transition p.m of the related Markov chain?

(b) what is the prob that he has lost his money at the end of 5 plays?

(c) what is the prob that the game lasts for more than 7 plays?

Sol:- Let x_n represent the amount with the player at the end of the n^{th} round of the play.

The state space of $x_n = \{0, 1, 2, 3, 4, 5, 6\}$

when the game is stopped, if the player loses 2, $x_n = 0$ & if he wins 4, $x_n = 6$ $(4+2=6)$

The TPM is

$$P = \begin{array}{c|c} & \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

0 and 6 states are called absorbing states
If entered he cannot change
any other state. (He stops playing)
initial probability distribution

$p^{(0)} = (0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0)$

$p^{(1)} = p^{(0)} P$

$= (0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0) P$

$= (0 \quad \frac{1}{2} \quad 0 \quad \frac{1}{2} \quad 0 \quad 0 \quad 0)$

$p^{(2)} = p^{(1)} P$

$= (0 \quad \frac{1}{2} \quad 0 \quad \frac{1}{2} \quad 0 \quad 0 \quad 0) P$

$p^{(2)} = (\frac{1}{4} \quad 0 \quad \frac{1}{2} \quad 0 \quad \frac{1}{4} \quad 0 \quad 0)$

$p^{(3)} = p^{(2)} P$

$= (\frac{1}{4} \quad \frac{1}{4} \quad 0 \quad 3/8 \quad 0 \quad 1/8 \quad 0)$

$p^{(4)} = p^{(3)} \cdot P = (3/8 \quad 0 \quad 5/16 \quad 0 \quad 1/4 \quad 0 \quad 1/16)$

$p^{(5)} = p^{(4)} P = (3/8 \quad 5/32 \quad 0 \quad 9/32 \quad 0 \quad 1/8 \quad 1/16)$

$n=0$

~~p(6)~~ $P(X_5 = 0) = 3/8$

(12)

$$p^{(6)} = p^{(5)} \, P$$

$$= \left(\frac{29}{64} \quad 0 \quad \frac{7}{32} \quad 0 \quad \frac{13}{64} \quad 0 \quad \frac{1}{8} \right)$$

$$p^{(7)} = p^{(6)} \cdot P$$

$$= \left(\underset{x=0}{\frac{29}{64}} \quad \underbrace{\frac{7}{64} \quad 0 \quad \frac{27}{128} \quad 0 \quad \frac{13}{128}} \quad \underset{x=6}{\frac{1}{8}} \right)$$

$$P[X_7 = 1, 2, 3, 4 \,\&\, 5]$$

$$\therefore \frac{7}{64} + 0 + \frac{27}{128} + 0 + \frac{13}{128}$$

$$= \frac{27}{64}$$

14) Classification of Markov Chains and States

- Write the classification of Markov chains and states.

Markov chain :

The transition probability matrix P has defined with the initial probabilities $\{p_j^{0}\}$ associated with the state E_j where $E_j = 0, 1, 2 \cdots n$ completely define a markov chain.

Markov chains are 2 types

(1) Ergodic (2) Regular.

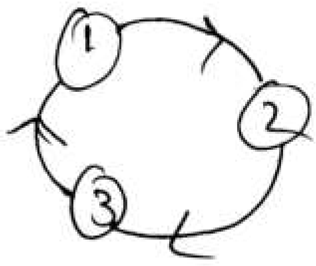
Ergodic : An ergodic markov chain the probability that it is possible to pass from one state to another state in finite no. of steps. Regarding of present state.

A regular Markov chain is defined as a chain having a transition matrix P such that for some power of P it has only non-zero positive probability values. Thus all the regular chains must be ergodic.

Irreduceible : A markov chain if all the states communicate to each other

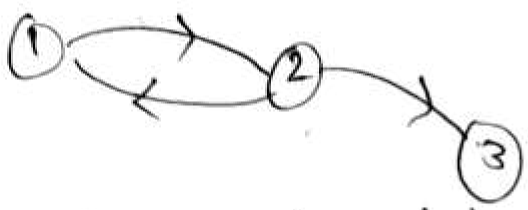
ex :

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & & & \\ 2 & & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} & \\ 3 & & & \end{array}$$



Recurrent state : come back to the state.

Eg :-



state 1 and state 2 are called recurrent states.

Transiant state : Not come back

the states 2 & 3 are called Transiant state.